NAG Toolbox for MATLAB

f07gt

1 Purpose

f07gt computes a diagonal scaling matrix S intended to equilibrate a complex n by n Hermitian positive-definite matrix A, stored in packed format, and reduce its condition number.

2 Syntax

```
[s, scond, amax, info] = f07gt(uplo, n, ap)
```

3 Description

f07gt computes a diagonal scaling matrix S chosen so that

$$s_j = 1/\sqrt{a_{jj}}.$$

This means that the matrix B given by

$$B = SAS,$$

has diagonal elements equal to unity. This in turn means that the condition number of B, $\kappa_2(B)$, is within a factor n of the matrix of smallest possible condition number over all possible choices of diagonal scalings (see Corollary 7.6 of Higham 2002).

4 References

Higham N J 2002 *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates whether the upper or lower triangular part of A is stored in the array **ap**, as follows:

uplo = 'U'

The upper triangle of A is stored.

uplo = 'L'

The lower triangle of A is stored.

Constraint: **uplo** = 'U' or 'L'.

2: **n – int32 scalar**

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

3: **ap**$(*)$ **– complex array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

The n by n Hermitian, packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i+j(j-1)/2)$ for $i \leq j$;
if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i+(2n-j)(j-1)/2)$ for $i \geq j$.

Only the elements of **ap** corresponding to the diagonal elements A are referenced.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **s**$(*)$ **– double array**

Note: the dimension of the array **s** must be at least $\max(1, \mathbf{n})$.

If **info** $= 0$ on exit, **s** contains the diagonal elements of the scaling matrix S.

2: **scond – double scalar**

If **info** $= 0$ on exit, **scond** contains the ratio of the smallest value of $\mathbf{s}(i)$ to the largest value of $\mathbf{s}(i)$. If **scond** ≥ 0.1 and **amax** is neither too large nor too small, it is not worth scaling by S.

3: **amax – double scalar**

$\max |a_{ij}|$. If **amax** is very close to overflow or underflow, the matrix A should be scaled.

4: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **ap**, 4: **s**, 5: **scond**, 6: **amax**, 7: **info**.

info > 0

If **info** $= i$, the ith diagonal element of A is not positive (and hence A cannot be positive-definite).

7 Accuracy

The computed scale factors will be close to the exact scale factors.

8 Further Comments

The real analogue of this function is f07gf.

9 Example

```
uplo = 'U';
n = int32(4);
ap = [complex(3.23, +0);
     complex(1.51, -1.92);
     complex(3.58, +0);
     complex(190000, +84000);
     complex(-23000, +111000);
     complex(40900000000, +0);
     complex(0.42, +2.5);
     complex(-1.18, +1.37);
     complex(233000, -14000);
     complex(4.29, +0)];
[s, scond, amax, info] = f07gt(uplo, n, ap)
```

```
s =
    0.5564
    0.5285
    0.0000
    0.4828
scond =
   8.8867e-06
amax =
   4.0900e+10
info =
           0
```